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An interactive possibilistic programming approach for a multi-objective hub location problem: Economic and environmental design

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Graphical Abstract

Proposed hub-and-spoke network with different transportation modes

Assimilation operator adopted in the proposed hybrid DE

Highlights

- Presenting a new multi-objective mathematical model for a $p$-hub location problem.
- Considering environmental aspects of noise pollution caused by vehicles.
- Considering different transportation modes.
- Applying an $Me$-based possibilistic programming method to cope with the uncertainty.
- Developing hybrid differential evolution and hybrid imperialist competitive algorithms to solve large-sized problems.

Abstract

This paper presents a new multi-objective mathematical model for a multi-modal hub location problem under a possibilistic-stochastic uncertainty. The presented model aims to minimize the total transportation and traffic noise pollution costs. Furthermore, it aims to minimize the maximum transportation time between origin-destination nodes to ensure a high probability of meeting the service guarantee. In order to cope with the uncertainties and the multi-objective model, we propose a two-phase approach, including fuzzy interactive multi-objective
programming approach and an efficient method based on the \( Me \) measure. Due to the \( NP \)-hardness of the presented model, two meta-heuristic algorithms, namely hybrid differential evolution and hybrid imperialist competitive algorithm, are developed. Furthermore, a number of sensitivity analyses are provided to demonstrate the effectiveness of the presented model. Finally, the foregoing meta-heuristics are compared together through different comparison metrics.

**Keywords:** Hub location problem; Environmental design; Uncertainty; Fuzzy interactive approach; Meta-heuristics.

1. Introduction

A hub location problem (HLP) is an extension of classical facility location problems, which is extensively implemented in airline systems, cargo delivery and telecommunication network design. In a hub-and-spoke network, there are a set of nodes (i.e., origins and destinations), where some of them are served as hub nodes and others are considered as non-hub nodes. In this network, there are two types of arcs: the first types of arcs are access arcs that make connections between non-hub nodes and hub-nodes, and the second types are hub arcs that connect the hubs nodes [1]. In the literature, several types of HLPs are developed, such as \( p \)-hub center, \( p \)-hub median and hub covering location problems [2]. In the \( p \)-hub location problems, a number of hub facilities are predefined and given by the decision makers. There are two primary assumptions in most of the HLPs as: (1) each pair of origin-destination (O-D) nodes have to be routed through at least one hub node, and (2) the network between hub nodes is a complete graph, where it means that there is a link between each pair of hub nodes [3, 4]. The interested readers are referred to Farahani, et al. [5] for more surveys on HLPs.

For many years, the main objective of the designed networks was to maximize the profit or minimize the total cost. Nowadays, sustainability is becoming a growing and interesting topic because of the concern about the environmental impacts of business activities [6]. In a network, transportation may cause different environmental impacts, such as resource consumption, toxic effects on ecosystems and humans, noise pollutions and greenhouse gas (GHG) emissions [7]. In the literature, most of the studies in the context of sustainability have focused on the reduction of GHG emissions; however, noise pollution is rarely discussed in the literature. Consequences of noise disturbance on human health have spotlighted the need to consider approaches that decrease the negative impacts of designed networks.

In the competitive world, customers are more sensitive to delivery requirement and choose a company with the lowest delivery time. In a hub-and-spoke network, regarding large volumes of packages transported between O-D nodes, the designed network needs to operate efficiently so as to be able to meet the service guarantees. Minimizing the longest path (i.e., maximum transportation time) between O-D nodes can provide an upper bound on the delivery times in a hub network [8]. Indeed, it returns a minimum time period needed to guarantee the service delivery. This minimum delivery time can be used to design time-constrained service offering to customers.

In the hub-and-spoke networks, shipments are not homogenous and have different service delivery requirements. In this respect, handling the large volumes of shipments needed to be done through different transportation modes. In the logistic networks, the most common transportation modes are road and rail transportation, where both of them differ in transit times [9]. Furthermore, transportation through air way recently becomes one of the popular transportation modes, since it has a shorter transit time in comparison to other transportation modes.

Due to the dynamic nature of a hub-and-spoke network, the respective costs, demands, distances, times and other parameters may change due to the uncertain circumstances. Since this probability of uncertainty will affect the design of the network, it is important to address uncertainties in the problem. In order to address the uncertainties, three types of modeling
techniques have been introduced in the literature, including stochastic programming, fuzzy programming and robust optimization [10].

In this paper, a new multi-objective mixed-integer non-linear mathematical programming (MOMINLP) model under possibilistic-stochastic uncertainty is introduced for a HLP to minimize (1) total investment costs (2) noise pollutions costs and (3) maximum transportation time between each pair of O-D nodes. The main contributions of this paper, which differentiate our effort from related studies, are as follows:

- Designing a new multi-objective mixed-integer non-linear mathematical model for a HLP that addresses the fluctuations in demand, costs, etc.
- Considering an objective function which tries to quantitatively model environmental aspects of noise pollutions caused by vehicles.
- Considering different transportation modes.
- Considering multi-capacity levels for hub nodes with different costs.
- Considering different types of vehicles for transportation between non-hub nodes and hub nodes and a number of trains and airplanes between hub-nodes.
- Applying a two-phase approach including an interactive fuzzy multi-objective programming approach and an efficient method based on Me measure to cope with the uncertainties and to deal with the multi-objective model.
- Developing two meta-heuristic algorithms to efficiently solve the large-sized instances in a reasonable amount of time.

The rest of the paper is organized as follows: Section 2 provides an overview of HLPs and reviews the related literature. Problem description and mathematical formulation are presented in Section 3. In Section 4, the proposed possibilistic programming is elaborated. Section 5 presents the solution method. Section 6 handles the computational experiments. Finally, the paper concludes in Section 7.

2. Literature review

This section presents a review of HLPs in the literature, including the prior studies in the area of HLPs, several studies that have focused on sustainable logistic network design and other studies that used different techniques to deal with the uncertainty in the HLPs.

In the area of HLP, the first known quadratic integer formulation of the hub location-allocation was presented by O’kelly [11]. Then, Campbell [12] simplified this model by developing a linear version of the presented model. There are other formulations of HLPs in the literature, for example [13-15].

There are numerous papers and reviews that considered green and sustainable logistics network design by focusing on issues such as CO₂ emissions, fuel choice, carbon intensity [16-20]. However, there are a limited number of quantitative studies concerning about optimizing the level of environmental pollution caused by transportation networks [21-23]. Noise pollution is an unwanted sound that has adverse effects such as physiological or psychological harm to human health and wellbeing [24]. Rahmani, et al. [24] presented two models for predicting in-city road-traffic noise pollution of one of the cities in Iran (Mashhad). They considered three parameters, including traffic volume, composition and speed. In order to solve the presented model, two genetic algorithms developed and the results demonstrated that there was unacceptable noise pollution due to the traffic in the city. Mohammadi, et al. [7] presented a sustainable hub location model which encompassed two environmental-based objective functions, including air and noise pollutions. The authors developed two meta-heuristic algorithms (i.e., SA and ICA) for solving the presented model.

Considering a capacity level for the hub nodes and adding a capacity constraint at the network design stage is one of the approaches that aims to control the congestion at the hub nodes that has been used by a number of studies [25-27].

HLPs under uncertainty have been studied in a limited number of articles. Sim, et al. [8] considered a stochastic p-hub center problem with variability in travel time which ensured a high probability of meeting the service-level. They assumed that the travel times are
independent and follow a normal distribution. Besides, they proposed a heuristic solution to solve the model. Contreras, et al. [28] studied an uncapacitated HLP in which uncertainty was associated with transportation costs and demands. A Mont-Carlo simulation-based algorithm combined with Benders decomposition algorithm was developed to solve the problem. A stochastic multi-objective hub covering problem is studied by Mohammadi, et al. [29]. In this study, a risk factor for transportation time was considered and it was assumed that the travel times follow a normal distribution. Besides, a new multi-objective imperialist competitive algorithm (MOICA) was developed to solve the presented model. Alumur, et al. [30] considered two sources of uncertainty in a HLP, including the set-up costs for the hub nodes and demand between each pair of O-D nodes. In another study, a stochastic-possibilistic programming approach was proposed to cope with the uncertain data in a HLP [7]. In their study, the capacity of hub nodes and most of the costs in the HLP were considered as uncertain parameters. Niakan, et al. [31] proposed a multi-objective optimization model for a HLP under uncertainty. In order to cope with the uncertainty, they adopted a hybrid approach, including an interval-valued fuzzy programming and a rough interval programming. They also used a multi-objective invasive weed optimization (MOIWO) meta-heuristic algorithm to solve the presented model.

A brief survey on the literature shows that, despite the importance of sustainability in the transportation networks, there is only one study which considered sustainability in the HLP in which they assumed that flows between the nodes are based on the vehicle unit. In this paper we relax this simplistic assumption by considering different types of transportation modes which makes the model more realistic. Besides, we use an efficient way (i.e., using an objective function to minimize the maximum transportation time) to ensure a high probability of meeting the service guarantee in the developed hub location model. Furthermore, the relevant literature does not sufficiently address the uncertainty in the HLPs. Hence, our study targets another gap in the existing literature by adopting an efficient method to capture different kinds of uncertainties in the input data.

3. Problem description and mathematical formulation

3.1. Modeling framework

This paper proposes a multi-objective single allocation hub location problem. Most of the previous studies consider classical objective function which aims to minimize the total transportation cost or total shipment time in the hub network. In this paper, we consider two new objective functions relating to noise pollution and maximum transportation time besides the classical cost function. Furthermore, in the presented model, we consider two types of transportation modes (i.e., rail mode and air mode) between each pair of hub nodes, and a road mode between non-hub nodes and hub nodes. The schematic view of the proposed hub-and-spoke network is depicted in Fig. 1.

{Please insert Fig. 1 here}

3.1.1. Noise pollution objective function

In this section, the considered noise pollution objective function is elaborated according to the following notations.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{10,k}$</td>
<td>Total noise level calculated at a reference distance of 10 meters from the nearside carriage way edge at hub node $k$ (hourly)</td>
</tr>
<tr>
<td>$L_{0,k}$</td>
<td>Basic noise level at hub node $k$ (hourly)</td>
</tr>
<tr>
<td>$\Delta f_k$</td>
<td>Adjustment of traffic flow at hub node $k$</td>
</tr>
<tr>
<td>$q_k$</td>
<td>Total flow at hub node $k$ (hourly)</td>
</tr>
</tbody>
</table>
According to [32], the total hourly noise level can be calculated as Eq. (1):

\[ L_{10,k} = L_{0,k} + \Delta f_k \]  

(1)

Furthermore, \( L_{0,k} \) and \( \Delta f_k \) are calculated as Eqs. (2) and (3), respectively.

\[ L_{0,k} = 42.2 + 10 \log_{10} q_k \text{ dB(A)} \]  

(2)

\[ \Delta f_k = 33 \log_{10} \left( s_k + 40 + \frac{500}{s_k} \right) + 10 \log_{10} \left( 1 + \frac{5p_k}{s_k} \right) - 68.8 \]  

(3)

where \( q_k \) and \( p_k \) are calculated as follows:

\[ q_k = f_k + \overline{OV}_k \]  

(4)

\[ p_k = \frac{100 f_k}{q_k} \]  

(5)

Mohammadi, et al. [7] considered the cost of noise pollution in a hub covering problem but for the purpose of simplicity, flows between the nodes were considered based on the vehicle unit. However, in our model, we assign a number of vehicles to each path between non-hub nodes and hub-nodes. Hence, the flow of vehicles to hub node \( k \) is presented as follows:

\[ f_k = \frac{\sum_{i} \sum_{v} A_{ik}^v T_{ik}}{\sum_{i} T_{ik} x_{ik}} \]  

(6)

By substituting Eqs. (2)-(6) and replacing the result into Eq. (1), \( L_{10,k} \) can be calculated as follows:

\[ L_{10,k} = 42.2 + 10 log_{10}(f_k + \overline{OV}_k) + 33 \log_{10} \left( s_k + 40 + \frac{500}{s_k} \right) + 10 \log_{10} \left( 1 + \frac{5}{s_k} \frac{100 f_k}{q_k} \right) - 68.8 \]  

(7)

Finally, the noise pollution cost at hub node \( k \) can be calculated as Eq. (8):

\[ NPC_k = \varphi \left[ \exp \left( \delta \left( L_{10,k} - L_{\text{max},k} \right) \right) - 1 \right] \]  

(8)

3.2. Mathematical programming

This section presents a new multi-objective mathematical model for a HLP according to the modeling framework of Section 3.1.

The main assumptions are explained as follows:

- Number of the established hub nodes is pre-defined.
- Single allocation assumption is considered.
There is a complete graph between the hub nodes.
- Capacity levels have been considered for each hub node.
- There are a limited number of vehicles available between non-hub nodes and hub nodes.
- Different transportation modes are considered between hub nodes.
- Transportation time between each pair of O-D nodes (i.e., using vehicles) follow an independent normal distribution.

**Sets:**
- \(i, j, k, l \in I\) Set of nodes
- \(t \in T\) Set of transportation modes
- \(n \in N\) Set of capacity levels available to hub \(k\)
- \(v \in V\) Set of different types of vehicles

**Parameters:**
- \(C_{ij}^v\) Transportation cost of a unit of the distance between nodes \(i\) and \(j\) using vehicle \(v\)
- \(F_{C_v}\) Fixed usage cost of vehicle \(v\)
- \(Ch_{kl}^t\) Transportation cost of a unit of the distance between hub nodes \(k\) and \(l\) using transportation mode \(t\)
- \(CS_k^n\) Cost of utilizing transportation mode \(t\) at hub \(k\)
- \(F_{C_k^n}\) Establishing cost of hub node \(k\) with capacity level \(n\)
- \(Th_{kl}^t\) Transportation time between hub nodes \(k\) and \(l\) using transportation mode \(t\)
- \(d_{kl}\) Distance between nodes \(i\) and \(j\)
- \(b_k^n\) Capacity of hub node \(k\) with level \(n\)
- \(B_{kl}^t\) Number of airplanes/trains between hub nodes \(k\) and \(l\)
- \(V_{C_v}\) Capacity of vehicle \(v\)
- \(MC_t\) Capacity of airplanes/trains in transportation mode \(t\)
- \(NV_v\) Maximum number of available vehicle \(v\)
- \(MC_t\) Maximum number of available airplanes/trains in transportation mode \(t\)
- \(P\) Predefined number of hub nodes that must be established
- \(W_{ij}\) Flow between each pair of O-D nodes

\[
O_i = \sum_j W_{ij}\]  Total amount of flow originating at node \(i\)
\[
D_i = \sum_j W_{ji}\]  Total amount of flow delivered at node \(i\)

**Decision variables:**
- \(Z_k^n\) 1; if hub node \(k\) is established with capacity level \(n\); otherwise, 0
- \(X_{ijkl}\) 1; if flow between nodes \(i\) and \(j\) transferred through hub nodes \(k\) and \(l\) using transportation mode \(t\); otherwise, 0
- \(X_{ik}\) 1; if there is a path between non-hub node \(i\) and hub node \(k\); otherwise, 0
- \(SL_k^n\) 1; if transportation mode \(t\) is served at hub \(k\); otherwise, 0
- \(\rho\) Maximum transportation time between each pair of O-D nodes

The multi-objective mathematical model is presented as follows:

\[
\text{Min } Z_1 = \sum_i \sum_k \sum_v A_{ik}^v (C_{ik}^v d_{kl} + F_{C_v}) X_{ik} + \sum_k \sum_l \sum_t B_{kl}^t Ch_{kl}^t d_{kl}
+ \sum_k \sum_t CS_k^n SL_k^t + \sum_k \sum_n F_{C_k^n} Z_k^n
\]  (9)
\[
\begin{aligned}
\text{Min } Z_2 &= \sum \sum \varphi \left| \exp \left( \left( 42.2 + 10 \log_{10}(f_k + MOV_k) \right) 
+ 33 \log_{10}\left( s_k + 4 + \frac{500}{s_k} \right) + 10 \log_{10} \left( 1 + \frac{5000 f_k}{s_k} \right) \right) - 68.8 \right| - L_{\max \cdot k} - 1 \right| Z_k^n \\
\text{Min } Z_3 &= \rho & (10) \\
\text{st.} \\
\sum_{i \neq k} X_{ik} + \sum_{n} Z_i^n &= 1 & \forall i & (12) \\
\sum_{k} Z_k^n &= P & (13) \\
\sum_{k} \sum_{l} \sum_{t} X_{ijkl}^t &= 1 & \forall i, j; i \neq j & (14) \\
\sum_{k} (O_i + D_j) X_{ik} \leq \sum_{n} b_k^n Z_k^n & \forall k & (15) \\
\rho \geq 0 & (11) \\
P\{T_{ik} + Th_{kl}^t + T_{lj} X_{ijkl}^t \leq \rho \} & \geq \gamma & \forall i, j, k, l, t & (16) \\
\sum_{k} VC_{i} A_{ik}^v & \geq \text{Max}(O_i, D_i) X_{ik} & \forall i, k; i \neq k & (17) \\
\sum_{k} MC_{i} B_{kl}^t & \geq \sum_{i} \sum_{j} \sum_{t} W_{ij} X_{ijkl}^t & \forall k, l; k \neq l & (18) \\
A_{ik}^v & \leq NV_i X_{ik} & \forall i, k, v & (19) \\
B_{kl}^t & \leq MV_i X_{kl} & \forall k, l, t & (20) \\
\sum_{t} X_{ijkl}^t & \geq X_{ik} + X_{jl} - 1 & \forall i, j, k, l; i \neq k, j \neq l & (21) \\
\sum_{l} X_{ijkl} & \leq X_{ik} & \forall i, j, k; i \neq k & (22) \\
\sum_{l} X_{ijkl} & \leq X_{jl} & \forall i, j, l; j \neq l & (23) \\
\sum_{t} X_{ijkl}^t & \leq \sum_{n} Z_k^n & \forall i, j, k & (24) \\
\sum_{t} X_{ijkl}^t & \leq \sum_{n} Z_l^n & \forall i, j, l & (25) \\
\sum_{n} Z_k^n & \leq 1 & \forall k & (26) \\
X_{ijkl}^t & \leq SL_k^t & \forall i, j, k, l, t & (27) \\
X_{ijkl}^t & \leq SL_l^t & \forall i, j, k, l, t & (28) \\
SL_k^t & \leq \sum_{n} Z_k^n & \forall k, t & (29) \\
SL_l^t & \leq \sum_{n} Z_l^n & \forall l, t & (30) \\
X_{ijkl}, X_{ik}, SL_k^t, Z_k^n & \in \{0, 1\} & \forall i, j, k, l, t & (31) \\
\rho \geq 0 & (31) \\
\end{aligned}
\]
The first objective function (9) minimizes total investment costs, including: (i) transportation costs between non-hub nodes and hub nodes, (ii) transportation costs between hub nodes through different transportation modes, (iii) costs of intermodal service at the hub nodes, and (iv) fixed costs of establishing hub nodes. The second objective function (10) measures the noise pollution costs. The third objective function (11) with constraints (16) and (31) minimize the maximum transportation time between each pair of O-D nodes.

Equation (12) indicates the single allocation assumption. Equation (13) assures that a total number of \( p \) hub nodes must be established. Equation (14) selects one hub pair \((k, l)\) for each O-D pair \((i, j)\) and a specific transportation mode \( t \). Constraint (15) ensures that the flow entering hub \( k \) does not exceed the capacity of hub node \( k \). Constraint (16) guarantees that the maximum transportation time between O-D nodes will be less than \( \rho \) with the probability of \( \gamma \). In this respect, there is at least a probability of \( \gamma \) that the total transportation time along the path \( ijkl \) is at most \( \rho \). Constraint (17) calculates the number of vehicles between non-hub nodes and hub-nodes. Constraint (18) calculates the number of assigned trains or airplanes between hub-nodes. Constraints (19) and (20) guarantee that the number of assigned vehicles between non-hub nodes and hub nodes and the number of assigned trains or airplanes between hub nodes do not exceed the maximum available number of them. Constraints (21)-(23) ensure the validation of path \( ijkl \) by checking the existence of links between nodes \( i \rightarrow k \) and nodes \( l \rightarrow j \). Constraints (24) and (25) obligate that if there is a path between nodes \( i, j, k \) and \( l \), nodes \( k \) and \( l \) must be located as hub nodes. Constraint (26) indicates that at most one capacity level must be selected for a hub node. Constraints (27) and (28) set the value of decision variables \( SL_k \) and \( SL_l \). Constraints (29) and (30) ensure that the assignment of a transportation mode between a pair of nodes is limited to considering the mentioned nodes as hub nodes. Constraint (31) is a domain constraint.

In order to transform the chance constraint (16) into a linear one, we use the following constraint:

\[
\rho \geq \left( T_{ik} + T_{hl} + T_{lj} + Z_{\gamma} \sqrt{\sigma_{ik}^2 + a_{ij}^2} \right) X_{ijkl} \quad \forall i, j, k, l, t \tag{32}
\]

where \( Z_{\gamma} \) is the \( z \)-value related to the 100 \( \gamma \)-th percentile from the standard normal distribution. It should be noted that the service level parameter \( \gamma \) is assumed to be at least 0.5.

### 3.3 Model validation

In order to validate the proposed model, we consider a small-sized test problem (i.e., \(|I| = 6, |P| = 2\)) and solved the presented multi-objective model as three single-objective problems, separately. The parameters are listed in Table 1. Notably, each single-objective problem was solved by the GAMS software 22.9 using Baron Solver. Fig. 2 depicts the result of the solved problems. In this figure, the hexagon and square nodes refer to the hub nodes and non-hub nodes, respectively. As it can be seen, different results have been obtained regarding to different objective functions. Therefore, it can be concluded that there is a conflict between three objective functions.

{Please insert Table 1 here}

{Please insert Fig. 2 here}

### 4. Crisp counterpart formulation

Generally, uncertainty in data can be classified into two categories, namely randomness and fuzziness [33]. Randomness stems from the random nature of data, in which the underlying action can be repeated many times. Since there are available and sufficient historical data in this class of uncertainty, a suitable discrete or continuous probability distribution can be adopted.
for each random data. Stochastic programming methods are the most applied approaches to cope with this sort of uncertainty. Fuzziness deals with imprecise parameters tainted with epistemic uncertainty, whose impreciseness arises from the lack of knowledge regarding the exact value of these parameters [6, 34]. In order to provide reasonable estimations for such imprecise parameters, we often have to rely on judgmental data based upon the experts’ experiences and professional feelings. Possibilistic programming approaches are usually applied to handle epistemic uncertainty in input data.

The proposed HLP is a possibilistic-stochastic multi-objective non-linear programming model. In the previous section, chance constraint (16) is transformed into a linear constraint (32). In our model, there is an epistemic uncertainty regarding exact values of some parameters (e.g., demands and transportation costs) due to unavailability of required data. In order to convert the possibilistic model into its equivalent crisp counterpart, we have adopted a two-phase approach. In the first phase, the proposed model is converted into an equivalent auxiliary crisp model by applying an efficient Me-based possibilistic programming method introduced by Xu and Zhou [35]. In the second stage, an interactive fuzzy multi-objective programming approach (TH method) developed by Torabi and Hassini [36] is adopted to find the preferred compromise solution.

4.1. Uncertain parameters

The uncertain parameters in the proposed mathematical model are tabulated in Table 2.

{Please insert Table 2 here}

4.2. Equivalent auxiliary crisp model

Several methods have been developed in the literature to transform a possibilistic model into an equivalent crisp one. The literature review demonstrates that the credibility-based possibilistic approaches, including the expected value [37], chance-constrained programming [38] and dependent chance-constrained programming [39] are the most applied approaches to handle epistemic uncertainty in input data. Among them, the possibilistic chance-constrained programming approach is the most applied one, in which the decision maker is able to control the satisfaction degree of chance constraints [40].

In the literature, several fuzzy measures have been introduced that can be used to transform a possibilistic constraint into its equivalent crisp one. The possibility (Pos) and necessity (Nec) are the basic fuzzy measures introduced in the literature. The Pos and Nec measures indicate the optimistic and pessimistic attitudes of the decision maker, respectively. Indeed, the Pos measure shows the possibility level of occurring an uncertain event that involves possibilistic parameters, while the Nec measure indicates the minimum possibility level of occurring an uncertain event. The credibility (Cr) is another fuzzy measure, which represents the certainty degree of occurring an uncertain event. In fact, the Cr measure can be defined as an average of the Pos and Nec measures [40]. Recently, Xu and Zhou [35] proposed a new fuzzy measure Me, which is an extension to the Cr measure. The main advantage of the Me measure over the other fuzzy measures is that it is more flexible to avoid extreme attitudes. That is, it can consider the combined attitude of the decision maker, which is something between optimistic and pessimistic views [34]. In the following, we introduce the concepts of possibility, necessity and credibility of a fuzzy event.

Let ξ be a fuzzy variable on possibility space ($\Theta, P(\Theta), Pos$). Then, its membership function is derived from the possibility measure Pos as follows:

$$\mu(x) = Pos\{\theta \in \Theta | \xi(\theta) = x\}, \quad x \in \mathbb{R}$$
Let $A$ be a set in $P(\Theta)$. Respectively, the necessity and credibility measures of $A$ are defined by:

$$\text{Nec}(A) = 1 - \text{Pos}(A^c)$$
$$\text{Cr}(A) = \frac{1}{2}(\text{Pos}(A) + \text{Nec}(A))$$

(34)  
(35)

The interested readers are referred to Liu [41] in order to see more concepts and properties of fuzzy theory.

In this paper, the $Me$-based possibilistic programming method is adopted to cope with the uncertain parameters of the proposed model. According to [35], the fuzzy measure $Me$ is defined as follows:

$$Me(A) = \text{Nec}(A) + \varepsilon(\text{Pos}(A) - \text{Nec}(A))$$

(36)

where $\varepsilon$ is the optimistic-pessimistic parameter to determine the combined attitude of the decision maker.

Consider the following possibilistic model with uncertain parameters:

$$\min \ f(x, \tilde{c})$$
$$\text{s.t.}$$
$$\tilde{A}x \geq \tilde{b}$$
$$\tilde{N}x \leq \tilde{d}$$
$$x \geq 0,$$

where $\tilde{c} = (\tilde{c}_1, \tilde{c}_2, ..., \tilde{c}_n)$, $\tilde{\Lambda} = [\tilde{a}_{ij}]_{m \times n}$, $\tilde{\Pi} = [\tilde{p}_{ij}]_{m \times n}$, $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, ..., \tilde{b}_n)^t$ and $\tilde{d} = (\tilde{d}_1, \tilde{d}_2, ..., \tilde{d}_n)^t$ indicate the triangular fuzzy numbers involve in the objective function and constraints.

In the $Me$-based possibilistic programming method, the expected value and chance-constrained operators based on the $Me$ measure are used to deal with the possibilistic objective functions and constraints, respectively. In this way, we can rewrite the model (37) as follows:

$$\min E[f(x, \tilde{c})]$$
$$\text{s.t.}$$
$$Me(\tilde{A}x \geq \tilde{b}) \geq \alpha$$
$$Me(\tilde{N}x \leq \tilde{d}) \geq \beta$$
$$x \geq 0,$$

where $\alpha$ and $\beta$ are the decision maker’s minimum confidence level for satisfaction of possibilistic constraints. Furthermore, $E$ is the expected value operator.

In the literature, there are several kinds of definitions for the expected value of a fuzzy variable (see for example, [37, 42-47]). Xu and Zhou [35] defined the expected value operator based on $Me$ measure as follows:

$$E[\xi] = \int_0^{+\infty} Me(\xi \geq r)dr - \int_{-\infty}^{0} Me(\xi \leq r)dr$$

(39)

where $\xi$ is a fuzzy variable on the possibility space $(\Theta, P(\Theta), \text{Pos})$.

Accordingly, the expected value of the triangular fuzzy variable $\xi = (\xi_1, \xi_2, \xi_3)$, when $\xi_1 \geq 0$, can be calculated by:

$$E[\xi] = \frac{(1 - \varepsilon)}{2} \xi_1 + \frac{1}{2} \xi_2 + \frac{\varepsilon}{2} \xi_3$$

(40)
According to [35], the discussed model (38) can be transformed into two approximation models namely, the upper approximation model (UAM) and the lower approximation model (LAM) that are presented as follows:

\[
\begin{align*}
\text{(UAM)} & \quad \min E[\tilde{\epsilon}]x \\
\text{s.t.} & \quad \text{Pos}\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\
& \quad \text{Pos}\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\
& \quad x \geq 0
\end{align*}
\]

and

\[
\begin{align*}
\text{(LAM)} & \quad \min E[\tilde{\epsilon}]x \\
\text{s.t.} & \quad \text{Nec}\{\tilde{A}x \geq \tilde{b}\} \geq \alpha \\
& \quad \text{Nec}\{\tilde{N}x \leq \tilde{d}\} \geq \beta \\
& \quad x \geq 0
\end{align*}
\]

The above possibilistic models can be transformed into two crisp equivalent linear models as Eqs. (43) and (44).

\[
\begin{align*}
\text{(UAM)} & \quad \min \left(\frac{1 - \epsilon}{2} C_{(1)} + \frac{1}{2} C_{(2)} + \frac{\epsilon}{2} C_{(3)}\right)x \\
\text{s.t.} & \quad A_{(2)}x + (1 - a)(A_{(3)} - A_{(2)})x \geq b_{(2)} - (1 - a)(b_{(2)} - b_{(1)}) \\
& \quad N_{(2)}x - (1 - \beta)(N_{(2)} - N_{(1)})x \leq d_{(2)} + (1 - \beta)(d_{(3)} - d_{(2)}) \\
& \quad x \geq 0
\end{align*}
\]

and

\[
\begin{align*}
\text{(LAM)} & \quad \min \left(\frac{1 - \epsilon}{2} C_{(1)} + \frac{1}{2} C_{(2)} + \frac{\epsilon}{2} C_{(3)}\right)x \\
\text{s.t.} & \quad A_{(2)}x - a(A_{(2)} - A_{(1)})x \geq b_{(2)} + (1 - a)(b_{(3)} - b_{(2)}) \\
& \quad N_{(2)}x + (1 - \beta)(N_{(3)} - N_{(2)})x \leq d_{(2)} - \beta(d_{(3)} - d_{(1)}) \\
& \quad x \geq 0
\end{align*}
\]

where \( \epsilon \) is the optimistic-pessimistic parameter.

Noteworthy, by solving the LAM and UAM models, the decision maker have the lower bound (i.e., solution obtained from the LAM model) and the upper bound (i.e., solution obtained from the UAM model) of the optimal decision. In this way, more information is provided to the decision maker when selecting the final solution [35].

Accordingly, the auxiliary crisp equivalent of the presented model with triangular fuzzy parameters is presented as follows:

**UAM:**

\[
\begin{align*}
\min Z^n = & \sum_i \sum_k \sum_l A''_l \left(\frac{1 - \epsilon}{2} C''_{ik(1)} + \frac{1}{2} C''_{ik(2)} + \frac{\epsilon}{2} C''_{ik(3)}\right) d_{il} + \left(\frac{1 - \epsilon}{2} FC''_{ik(1)} + \frac{1}{2} FC''_{ik(2)} + \frac{\epsilon}{2} FC''_{ik(3)}\right) X_{il} \\
& + \sum_i \sum_j \sum_l \left(1 - \epsilon\right) CS'_{ij(k)} + \frac{1}{2} CS'_{ij(k)} + \frac{\epsilon}{2} CS'_{ij(k)}\right) d_{ij} \right) X_{il}
\end{align*}
\]

**LAM:**

\[
\begin{align*}
\min Z^n = & \sum_i \sum_k \sum_l A''_l \left(\frac{1 - \epsilon}{2} C''_{ik(1)} + \frac{1}{2} C''_{ik(2)} + \frac{\epsilon}{2} C''_{ik(3)}\right) d_{il} + \left(\frac{1 - \epsilon}{2} FC''_{ik(1)} + \frac{1}{2} FC''_{ik(2)} + \frac{\epsilon}{2} FC''_{ik(3)}\right) X_{il} \\
& + \sum_i \sum_j \sum_l \left(1 - \epsilon\right) CS'_{ij(k)} + \frac{1}{2} CS'_{ij(k)} + \frac{\epsilon}{2} CS'_{ij(k)}\right) SL_{ij}
\end{align*}
\]

where \( \epsilon \) is the optimism-pessimism parameter.
\[
\begin{align*}
\text{Min } Z_2 &= \sum_{i=1}^{n} \sum_{k=1}^{p} \phi \exp \delta \left[ 42.2 + 10 \log_{10} \left( f_k + \frac{1}{2} \text{MOV}_{ik}^{(1)} + \frac{1}{2} \text{MOV}_{ik}^{(2)} + \frac{\varepsilon}{2} \text{MOV}_{ik}^{(3)} \right) \right] \\
&+ 33 \log_{10} \left( \frac{1}{2} s_{i(1)} + \frac{1}{2} s_{i(2)} + \frac{\varepsilon}{2} s_{i(3)} \right) + 4 + \frac{500}{\left( \frac{1}{2} s_{i(1)} + \frac{1}{2} s_{i(2)} + \frac{\varepsilon}{2} s_{i(3)} \right)} - L_{\text{max},k} - 1 Z_i^* \\
&+ 10 \log_{10} \left( f_k + \frac{1}{2} \text{MOV}_{ik}^{(1)} + \frac{1}{2} \text{MOV}_{ik}^{(2)} + \frac{\varepsilon}{2} \text{MOV}_{ik}^{(3)} \right) \\
&- 68.8
\end{align*}
\]

\[
\text{Min } Z_3 = \rho
\]
\[
\text{s.t.} \quad \sum_{i=1}^{n} \left[ \left( O_{i(2)} - (1-\alpha)(O_{i(2)} - O_{i(1)}) \right) + \left[ D_{i(2)} - (1-\alpha)(D_{i(2)} - D_{i(1)}) \right] \right] X_{ik} \leq \sum_{n} \left[ b_{k(1)}^n + (1-\alpha)(b_{k(3)}^n - b_{k(2)}^n) \right] Z_k^n
\]
\[
P\left( T_{ik} + \left( T_{ik}^{(1)} - (1-\alpha)(T_{ik}^{(2)} - T_{ik}^{(1)}) \right) \right) X_{ijkl}^* \leq \rho \geq \gamma
\]
\[
\sum_{i} \text{VC}_i \alpha_i \geq \text{Max} \left[ \sum_{i} \left[ O_{i(2)} - (1-\gamma)(O_{i(2)} - O_{i(1)}) \right] + \left[ D_{i(2)} - (1-\alpha)(D_{i(2)} - D_{i(1)}) \right] \right] X_{ik}
\]
\[
\sum_{i} \text{MC}_i \beta_i \geq \sum_{i} \sum_{j} \left[ W_{ij(2)} - (1-\gamma)(W_{ij(2)} - W_{ij(1)}) \right] X_{ijkl}^*
\]
\[
\text{Other constraints.}
\]

LAM:
\[
\text{Min } \text{E}[Z_1]
\]
\[
\text{Min } \text{E}[Z_2]
\]
\[
\text{Min } Z_3 = \rho
\]
\[
\text{s.t.} \quad \sum_{i=1}^{n} \left[ \left( O_{i(2)} + (1-\alpha)(O_{i(3)} - O_{i(2)}) \right) + \left[ D_{i(2)} + (1-\alpha)(D_{i(3)} - D_{i(2)}) \right] \right] X_{ik} \leq \sum_{n} \left[ b_{k(2)}^n - (\alpha)(b_{k(3)}^n - b_{k(2)}^n) \right] Z_k^n
\]
\[
P\left( T_{ik} + \left( T_{ik}^{(1)} + (1-\alpha)(T_{ik}^{(3)} - T_{ik}^{(2)}) \right) + T_{ij} \right) X_{ijkl}^* \leq \rho \geq \gamma
\]
\[
\sum_{i} \text{VC}_i \alpha_i \geq \text{Max} \left[ \sum_{i} \left[ O_{i(2)} + (1-\alpha)(O_{i(3)} - O_{i(2)}) \right] + \left[ D_{i(2)} + (1-\alpha)(D_{i(3)} - D_{i(2)}) \right] \right] X_{ik}
\]
\[
\sum_{i} MC_{i} B^{i}_{j} \geq \sum_{i} \sum_{j} \sum_{ij} \left[ W_{ij}^{(2)} + (1-\gamma)(W_{ij}^{(3)} - W_{ij}^{(2)}) \right] X_{ijkl}^{i}
\]  
(58)

Other constraints.

4.3. Fuzzy interactive programming approach

Among several approaches that have been developed to solve the multi-objective crisp models, fuzzy interactive methods are one of the most attractive approaches that are increasingly applied in the recent studies [7, 40]. The main advantage of these methods is the ability in measuring and adjusting the satisfaction degree of each objective function [36]. In this paper, a hybrid solution approach is utilized to cope with the uncertainty and to deal with the multi-objective mathematical model, which is the combination of presented possibilistic programming and TH method. The steps of the proposed hybrid solution approach can be summarized as follows:

**Step 1**: Convert the possibilistic parameters of the objective functions to their respective crisp equivalents using \( Me \) measure.

**Step 2**: Convert the possibilistic constraints into their respective crisp equivalents using \( Me \) measure.

**Step 3**: Determine the positive ideal solution (PIS) and the negative ideal solution (NIS) for each objective function (i.e., for both LAM and UAM models).

**Step 4**: Determine a linear membership function for each objective function (i.e., for both LAM and UAM models) as Eq. (59).

\[
\mu_{k}(z) = \begin{cases} 
1 & \text{if } z_{k} < z_{k}^{PIS} \\
\frac{z_{k}^{NIS} - z_{k}}{z_{k}^{NIS} - z_{k}^{PIS}} & \text{if } z_{k}^{PIS} \leq z_{k} \leq z_{k}^{NIS} \\
0 & \text{if } z_{k} > z_{k}^{NIS}
\end{cases}
\]  
(59)

**Step 5**: Convert the equivalent crisp multi-objective model into a single-objective model using the TH aggregation function (i.e., for both LAM and UAM models). It should be noted that the TH method ensures to obtain the efficient solutions [36]. The TH aggregation function is computed as follows:

\[
\text{Max } \psi(x) = \vartheta \lambda_{0} + (1-\vartheta) \sum_{k} \varphi_{k} \mu_{k}(z) \]  
(60)

\[\text{st.} \]

\[\lambda_{0} \leq \mu_{k}(z) \quad k = 1, 2, 3 \]  
(61)

\[x \in F(x) \quad \lambda_{0}, \varphi \in [0,1] \]  
(62)

where \( F(x) \) denotes the feasible region. Furthermore, \( \vartheta \) and \( \varphi_{k} \) (\( \sum_{k} \varphi_{k} = 1 \)) are the coefficient of compensation and the importance of the \( k \)-th objective function, respectively. In this way, the decision makers can reach a compromise solution between the minimum of the objective functions and the weighted sum of the objective functions.

**Step 6**: Solve the equivalent crisp single-objective model given the coefficient of compensation (\( \vartheta \)) and the relative importance of the objective functions (\( \varphi_{k} \)). If the decision maker is satisfied with the obtained solution, stop; otherwise, change the values of mentioned parameters in order to obtain another compromise solution.

5. Solution method

The presented model in Section 3 is a mixed integer nonlinear programming (MINLP) model. Several optimization techniques have been developed to solve these problems such as
branch-and-bound, branch-and-price and branch-and-cut. However, solving such an MINLP problem is very computationally challenging, especially for large-sized instances.

In the presented model, test problems up to ten nodes and five hub nodes are successfully solved using BARON solver, while attempts to solve the large-sized instances (i.e., more than ten nodes) were unsuccessful due to the huge computation time. To address this challenge, we develop two efficient meta-heuristic algorithms (i.e., hybrid differential evolution (HDE) and hybrid imperialist competitive algorithm (HICA)) in order to find near optimal solutions in a reasonable amount of time. Notably, the convexity of non-linear objective function (10) is proved in Section 5.1.

5.1. Proof of convexity

As the presented mathematical model must find the optimal solution of the original problem, it is needed to prove that the non-linear objective function (10) is convex.

**Lemma 5.1.1.** Objective function (10) is convex.

**Proof.** Objective function (10) is written in exponential form, and we know that the an exponential function $e^x$ is strictly increasing and convex [48]. Therefore, it can be concluded that the objective function (10) is convex.

5.2. Meta-heuristic algorithms

In this section, at first, we explain the solution representation and the adopted method for handling the constraints. Afterward, two developed meta-heuristic algorithms are briefly explained.

5.2.1. Solution representation

Deciding how to represent the solution in the search space is one of the important steps in designing a meta-heuristic algorithm because it has a great influence on the performance of the algorithm. In this article, we use a continuous representation solution proposed by Mohammadi, et al. [29]. The presented hub location model contains three different parts including location-allocation (LA), transportation mode selection (TMS) and capacity level selection for hub nodes (CLS). For the LA part we refer to continuous representation solution proposed by Mohammadi, et al. [29]. We briefly explain the other parts of the solution representation in the following sections.

5.2.1.1. Transportation mode selection (TMS)

The TMS part of the proposed solution representation includes two ($n \times n$) matrices, where $n$ is the number of nodes. The first matrix related to the TMS between the non-hub nodes and hub nodes, filled with random numbers belonging to $(1, |v|)$. Similarly, the second matrix related to the TMS between hub nodes, filled with random numbers belonging to $(1, |t|)$. For instance, consider a hub network with five nodes, two types of vehicles and two transportation modes between hub nodes ($|n| = 5, |v| = 3, |t| = 2$). Figs. 3 depicts the TMS part between the non-hub nodes and hub nodes, and Fig. 4 depicts the TMS part between hub nodes. As it can be seen, vehicle type one is assigned between non-hub node two and hub node three (i.e., if node three is selected as a hub node). Furthermore, transportation mode two is selected between hub nodes two and three (i.e., if nodes two and three are selected as hub nodes).

{Please insert Fig. 3 here}

{Please insert Fig. 4 here}
5.2.1.2. Capacity level selection (CLS)

The CLS part of the solution representation contains an \((1 \times n)\) array, in which each bit corresponds to a random integer number belonging to \((1, |N|)\). It should be noted that \(|N|\) is the number of available capacity levels. A sample array with five nodes and three available capacity levels for hub nodes (i.e., \(|n| = 5, |N| = 3\)) is shown in Fig. 5. As it can be seen, if hub node 3 is selected as a hub node, it should be established with capacity level one.

5.2.2. Handling the constraints

Since the presented mathematical model comprises several constraints, the developed algorithms should be able to handle them during optimization. The adopted solution representation can guarantee the feasibility of all the constraints, except Constraint (15), which indicates the capacity limit. In the literature, the most popular approach to handle these sorts of constraints is to use penalty functions. In this method, infeasible solutions are penalized by reducing their fitness values in proportion to their degree of constraint violation \([49, 50]\).

In this paper, we employ a penalty function introduced by Yeniay \([51]\). Hence, at first, the penalty value of an infeasible constraint \((g(x) \leq d)\) is calculated by:

\[
P(x) = R \times \text{Max}\left\{\left(\frac{g(x)}{d} - 1\right), 0\right\}
\]

(63)

where \(P(x)\) and \(R\) indicate the penalty value and a large number, respectively.

Afterward, the penalty values are considered for all of the three objective functions through an additive function given below.

\[
F(x) = \begin{cases} 
  f(x) & x \in \text{feasible region} \\
  f(x) + P(x) & x \notin \text{feasible region}
\end{cases}
\]

(64)

5.2.3. Imperialist competitive algorithm (ICA)

Imperialist competitive algorithm (ICA), which belongs to the class of evolutionary algorithms (EAs), mimics the process of socio-political evolution of human \([52]\). This algorithm starts with an initial population which is called country. Afterward, some of the best countries with the least cost are selected to be imperialist and the rest of them are distributed among the aforementioned imperialists based on their power \([53]\). Thus, imperialists and their colonies together create the initial empires. In the next step, similar to the assimilation policy which was perused by some of the imperialist states, colonies start moving toward their relevant imperialist. Afterward, a competition begins among all the empires. The total power of an empire depends on both the power of the imperialist and the power of its colonies. Any empire that is not able to succeed in this competition and increase its power will be eliminated. Therefore, weaker empires will gradually lose their power and finally they will collapse. Finally, during the competition among empires and also the collapse mechanism, all the countries will converge to a stage, where just one empire exists and all the other countries become colonies of that empire.

Fig. 6 depicts the assimilation operator in detail. In order to move colonies toward imperialists in other direction, a random amount of deviation is considered. Usually \(\theta\) is a random number and follows a uniform distribution. It should be noted that \(d\) is the distance between colony and imperialist.
5.2.3.1. Hybrid imperialist competitive algorithm (HICA)

In order to prevent the algorithm from becoming stuck at a local minimum, an extra operator is added to the ICA, namely revolution. In the developed HICA algorithm, a number of colonies randomly selected to undergo the revolution operator. Inversion and reversion methods are adopted for the continuous and binary parts, respectively. Fig. 7 shows the steps of the proposed HICA algorithm.

{Please insert Fig. 7 here}

5.2.4. Differential evolution (DE)

Differential evolution (DE) is a simple and efficient heuristic introduced by Storn and Price [54]. Similar to other EAs, DE starts with an initial population which is chosen randomly over the variable domain [55]. In this algorithm, mutation and crossover operators are used to evolve the initial population. After initialization, a mutation operator is employed to produce a mutant vector with respect to each individual, so-called target vector, as follows:

\[
MV_i = \omega v_1 + F(\omega v_2 - \omega v_3)
\]  

(65)

where \( v_1, v_2 \) and \( v_3 \) are the mutually different random indexes that are not equal to \( i \). Furthermore, \( F \) is the amplification coefficient (i.e. \( F \in [0, 2] \)).

In the next step, a crossover operator is utilized to increase the diversity of the algorithm. By using the crossover operator, each mutated vector shares its information with another predetermined (target) vector to create new solution vector \( \mu_i = \{\mu_{i1}, \ldots, \mu_{ik}, \ldots, \mu_{in}\} \) as Eq. (66).

\[
\mu_{ik} = \begin{cases} 
MV_{ik}, & \text{if } \text{rand}(k) \leq CR \text{ and } k = \text{rnbr}(i) \\
\omega_{ik}, & \text{if } \text{rand}(k) > CR \text{ and } k \neq \text{rnbr}(i)
\end{cases}
\]

(66)

where \( \text{rand}(k) \) is the \( k \)-th element of an \( N \)-dimensional uniform random number \( \in [0, 1] \), \( CR \) is the crossover rate and \( \text{rnbr}(i) \) is a randomly chosen index \( \in \{1, \ldots, N\} \) which guarantees that at least one mutated dimensional value is used in the new created vector. Finally, if the new created vector yields a reduction in the value of the fitness function, it replaces the target vector in the following generation.

5.2.4.1. Hybrid differential evolution (HDE)

In this section, we propose the hybrid DE (HDE) algorithm to obtain better solutions in comparison with the classical DE. In the literature, several efforts have been done to improve the performance of DE. Among them, a significant number of studies tried to introduce new mutation and crossover operators for generating better solutions (i.e., see [56-59]). Generally, there are five most frequently referred mutation strategies as shown in Table 3 [58].

{Please insert Table 3 here}
\[ \mu_{ik} = \begin{cases} MV_{ik}, & \text{for } k = \langle l \rangle_D, \langle l + 1 \rangle_D, \ldots, \langle l + L - 1 \rangle_D \\ \omega_{ik}, & \text{otherwise} \end{cases} \]  

(67)

where \( \langle l \rangle_D \) is the modulo function with modulus \( D \), and \( l \) is a randomly chosen index \( \in \{1, \ldots, N\} \).

Moreover, integer \( L \) can be calculated as \( P_r(L \geq v) = CR^{v-1}, v > 0 \).

In the literature, several studies have used only one mutation and crossover operators to search the solution space. However, in the proposed HDE, all the aforementioned mutation and crossover operators are used randomly. Furthermore, the assimilation operator, explained in Section 5.2.3, is applied on the mutated vector to improve the ability of the algorithm in exploration and exploitation. Similar to Fig. 6, the mutated vector (i.e., instead of colony) is moved toward an auxiliary vector (i.e., instead of imperialist) with a random angle. Notably, the auxiliary vector can be selected through the roulette wheel selection method or it can be the best vector among the whole population. Fig. 8 shows the pseudo code of the proposed HDE.

6. Computational experiments

In this section, several sensitivity analyses are performed on a number of small- and medium-sized test problems to validate the correctness of the proposed model. Afterward, the performance of the developed algorithms (i.e., HDE and HICA) is evaluated. Notably, the presented model is coded in GAMS 22.9 software and solved using BARON solver. All the computations are performed on a Pentium 4 computer with 2.66 GHz CPU and 4 GB of RAM.

6.1. Sensitivity analyses

In this section, several sensitivity analyses are carried out to investigate the influence of parameters variations on the objective functions. At first, objective function values for the LAM and UAM under different values of \( \alpha \) and \( \varphi_k \) are reported in Tables 4 and 5, respectively. Afterward, objective function values of the LAM and UAM under different values of \( \vartheta \) are reported in Table 6. Noteworthy, since the optimistic attitude of the decision maker is considered in the UAM, the feasible region of the UAM is larger than that of the LAM. Therefore, the UAM gives better solutions.

Figs. 9 and 10 illustrate the changes in \( OFV_1 \) and \( OFV_2 \) versus the changes of the mean of flow, respectively. As it can be seen, all of the objective functions are getting worse as an increase in the mean of flow. Indeed, by increasing the mean of flow, the model is forced to use more vehicles and airplanes or trains to cover the demands, and also hub nodes with higher capacity must be established. As a result, \( OFV_1 \) will be increased. On the other hand, increasing the mean of flow will increase the flow of heavy vehicles \( (f_k) \), and as a consequence, \( OFV_2 \) will be increased.
Figs. 11 and 12 show the changes of OFV\textsubscript{1} and OFV\textsubscript{3} versus the changes of \(Ch_{kl}\), respectively. Two transportation modes are considered between hub nodes, where \(t=1\) and \(t=2\) stand for air mode and rail mode, respectively. Generally, the transit time of air mode is less than that of rail road while the transfer cost of air mode is more than that of rail mode. As it can be seen, when the mean of \(Ch_{kl}\) is increased from 30 to 50, the model decides to use air mode between most of the hub nodes (i.e., OFV\textsubscript{1} is between [442051, 469504] and OFV\textsubscript{3} is between [123.3, 124.2]). However, for values of the mean of \(Ch_{kl}\) more than 50, it is preferred to choose rail mode instead of air mode between most of the hub nodes (i.e., OFV\textsubscript{1} is between [356729, 354212] and OFV\textsubscript{3} is between [144, 144.9]).

6.2. Comparative study

In this section, the performance of the developed meta-heuristic algorithms is evaluated in a comparative study over a number of randomly generated data sets. The generated data sets contain 30 test problems ranging from small- to large-sized instances (up to 100 nodes). A response surface methodology (RSM) is used to determine the tuned parameters of HDE and HICA. Table 7 represents the tuned parameter of HDE and HICA.

Three important comparison metrics are considered to investigate the significance of the difference between two developed meta-heuristic algorithms and they are summarized as follows:

- Average convergence (%)

It can be defined as the average of convergence rate of solutions [60], which can be calculated by:

\[ACR = 1 - \frac{\text{Average fitness} - \text{Optimal fitness}}{\text{Optimal fitness}} \times 100\]  
\[(68)\]

where the average fitness is the average fitness value of solutions, and the optimal fitness is the optimal value of the corresponding instance.

- Convergence diversity (%)

It can be defined as the difference between the convergence rate of the best and worst fitness values of the corresponding instance [60, 61]. It can be presented by:

\[\text{CD} = CR_{\text{Best Fitness}} - CR_{\text{Worst Fitness}}\]  
\[(69)\]

where \(CR_{\text{Best Fitness}}\) and \(CR_{\text{Best Fitness}}\) are the convergence rate of the best and worst fitness values, respectively.

- Computation time

It is defined as the average required time to find an acceptable near-optimal solution.
Tables 8 and 9, show the experimental results on HDE and HICA for instances up to ten nodes and five hub nodes, respectively. The percentage of convergence rates for the five replications of each test problem are averaged and recorded as the “ACR” column. The values in the “CD” column indicate the convergence diversity of test problems. Furthermore, the average computation time of each test problem is reported in the “computation time” column.

{Please insert Table 8 here}

{Please insert Table 9 here}

Several statistical tests are carried out to see whether a significant difference exists between the performance of HDE and HICA in terms of ACR, CD and computation time. Noteworthy, there are two types of statistical tests, namely parametric and non-parametric tests. In order to use parametric tests, it is necessary to check three conditions, including independence, normality and heteroscedasticity [62]. In our case, the independence condition can be met because the initial population of each algorithm is generated randomly. In the following, the normality and heteroscedasticity analyses are carried out using the Kolmogorov-Smirnov and Levene tests, respectively. Table 10 presents the p-values obtained by the Kolmogorov-Smirnov test. The p-values obtained by the Levene test are also reported in Table 11.

{Please insert Table 10 here}

{Please insert Table 11 here}

As shown in Table 10, normality is not fulfilled in the CD metric (i.e., the p-value is less than 0.05). Similarly, Table 11 shows that there is a significant difference between the variances of HDE and HICA in terms of the CD metric. Accordingly, a non-parametric test namely Wilcoxon test is used to compare HDE and HICA in terms of the CD metric. However, a parametric test namely paired t test is used in other cases. The analytical results of the paired t-test and Wilcoxon test are provided in Tables 12 and 13, respectively.

According to Table 12, the p-value related to average convergence rate is less than 0.05. Hence, it can be concluded that there is a significant difference between HDE and HICA in terms of the ACR metric. However, the results show that there is no significant difference between HDE and HICA in terms of the computation time. As it can be seen in Table 13, the p-value of Wilcoxon test is less than 0.05. Hence, there is a significant difference between HDE and HICA in terms of the CD metric.

{Please insert Table 12 here}

{Please insert Table 13 here}

Since the presented mathematical model is a NP-hard, large-sized test problems cannot be solved by GAMS software. In this respect, for large-sized test problems, a gap between HDE and HICA algorithms is presented through the percentage of relative gap (PRG) measure [63] calculated as $\left[100 \times (S_{HDE} - S_{HICA})/S_{HICA}\right]$, where $S_{HDE}$ and $S_{HICA}$ are the obtained solutions from HDE and HICA, respectively. The comparison results between HDE and HICA are reported in Table 14. In this table, the PRGs for the five replications of each test problem are averaged.
and recorded as the "APRG (%)" column. Furthermore, the required computation time for the HDE and HICA are also given in the "Time(s)" column. It should be noted that in all test problems HDE performs better than the proposed HICA in terms of the PRG measure. As shown in Table 14, the minimum APRG between HDE and HICA is 3.1%. Besides, the maximum APRG between HDE and HICA is 14.5%. The APRG between HDE and HICA, and the corresponding average computation times are shown in Figs. 13 and 14, respectively.

Regarding the similarity in the computation time and outperformance of HDE comparing to HICA in both small and large-sized instances, it can be concluded that HDE has better performance than HICA in solving the presented hub location model.

{Please insert Table 14 here}

{Please insert Fig. 13 here}

{Please insert Fig. 14 here}

7. Conclusion

In this paper, a new possibilistic-stochastic multi-objective mathematical model for a $p$-hub location problem was presented in order to minimize the total transportation cost, traffic noise pollution cost and the maximum transportation time between origin-destination nodes. In the presented model, cost benefits can be achieved through the use of different transportation modes and establishing hub nodes with different capacity levels. This study was the first attempt to model and solve the hub location problem under uncertainty using a hybrid two-phase solution approach, including efficient $Me$-based possibilistic programming and an interactive fuzzy multi-objective programming (i.e., TH method). The results demonstrated that the proposed hybrid approach can efficiently capture the epistemic uncertainty and deal with the multi-objective model. Furthermore, several sensitivity analyses are carried out to investigate the influence of parameters variations on the objective functions. Finally, in order to solve the large-sized instances, two meta-heuristic algorithms, namely HDE and HICA, were developed and their performance was evaluated through different comparison metrics. The results showed the superiority of the developed HDE in comparison with HICA.

Including social aspects in the network design decisions can help us to better evaluate the impact of the designed network on its stakeholders, such as employees, customers and local communities. Therefore, a possible extension to the current research is to propose a quantitative optimization model to address social responsibility aspect of sustainability in designing a hub-and-spoke network.

References


Figures

Fig. 1. Proposed hub-and-spoke network.
Fig. 2a. Optimal solution considering $OFV_1$.

Fig. 2b. Optimal solution considering $OFV_2$.

Fig. 2c. Optimal solution considering $OFV_3$. 
Fig. 3. Transportation mode assignment between the non-hub nodes and hub nodes.

Fig. 4. Transportation mode assignment between the hub nodes.
Fig. 5. Capacity level assignment for hub nodes.
Fig. 6. Assimilation operator with a random angle.
Set the parameters \((N_{pop}, N_{emp}, I_t, R_p)\)

\(Iter=0\)

Generate initial countries (Randomly) \(\leftarrow N_{pop}\)

Evaluate fitness of each country

From initial countries:

\(a)\) Choose most powerful countries as the imperialists \(\leftarrow N_{emp}\)

\(b)\) Assign other countries to imperialists based on imperialist power

\(terminate \leftarrow false\)

\(While\) (\(terminate = false\)) \(do\)

Move the colonies toward their relevant imperialist (Assimilation)

Do revolution among the colonies

Evaluate fitness of each colony

\(If\) there is a colony in an empire which has lower cost than the imperialist \(then\)

Exchange the position of that imperialist and colony

\(End\ if\)

Compute the total cost of all empires

Select the weakest colony from the weakest empire and assign it to the strongest empire

\(If\) there is an empire with no colonies \(then\)

Eliminate this empire

\(End\ if\)

\(If\) \((Iter \geq I_t)\) \(then\)

\(terminate \leftarrow true\)

\(End\ if\)

\(Iter=Iter+1\)

\(End\ while\)

\(Fig.\ 7.\) Pseudo code of the HICA.
Set the parameters \((N_{pop}, It, CR)\)

\[\text{Iter}=0\]

Create a random initial population \(\leftarrow N_{pop}\)

Evaluate fitness of each individual

\[\text{terminate} \leftarrow \text{false}\]

\textbf{While} (\text{terminate} = \text{false}) \textbf{do}

\textbf{For} \(j=1 : N_{pop}\)

Select parents (Binary Tournament Selection)

Apply one of the mutation operators, randomly

a) Rand/1
b) Best/1
c) Current to best/1
d) Best/2

Select an auxiliary vector (Roulette Wheel Selection)

Move the mutated vector toward the auxiliary vector (Assimilation)

Apply one of the crossover operators, randomly

a) Binomial
b) Exponential

If fitness\(_{\text{new vector}}\) < fitness\(_{\text{target vector}}\) then

Replace the target vector by the new vector

\textbf{End if}

\textbf{End For}

If (\text{Iter} \geq \text{It}) then

\[\text{terminate} \leftarrow \text{true}\]

\textbf{End if}

\[\text{Iter} = \text{Iter} + 1\]

\textbf{End while}

\textbf{Fig. 8.} Pseudo code of the HDE.
Fig. 9. OFV$_1$ vs. the mean of flow.
Fig. 10. OFV$_2$ vs. the mean of flow.
Fig. 11. OFV$_1$ vs. the mean of transportation cost between hub nodes using transportation mode 1.
Fig. 12. OFV$_3$ vs. the mean of transportation cost between hub nodes using transportation mode 1.
Fig. 13. APRG between HDE and HICA.
Fig. 14. Computation time of HDE and HICA.
Table 1. Parameters range for test problems.

<table>
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<th>Parameters</th>
<th>Values</th>
<th>Parameters</th>
<th>Values</th>
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<td>$b_k^n$</td>
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<td>$\sim$ Uniform(10, 30)</td>
<td>$VC_v$</td>
<td>$\sim$ Uniform(20, 40)</td>
</tr>
<tr>
<td>$CH_{kl}$</td>
<td>$\sim$ Uniform(20, 40)</td>
<td>$MC_t$</td>
<td>$\sim$ Uniform(50, 100)</td>
</tr>
<tr>
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<td>$NV_v$</td>
<td>$\sim$ Uniform(10, 20)</td>
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<td>$CS_k$</td>
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<td>$MV_t$</td>
<td>$\sim$ Uniform(5, 8)</td>
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<td>$F^n_k$</td>
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<td>$\sim$ Uniform(40, 80)</td>
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<td>$T_{ij}$</td>
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<td>$\bar{OV}_k$</td>
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<td>$d_{kl}$</td>
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<td>$\sigma_{ij}$</td>
<td>$\sim$ Uniform(0, 2)</td>
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<td>( C_{hkl} )</td>
<td>( FC_v )</td>
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<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td></td>
<td>( (C_{ij1}^v, C_{ij2}^v, C_{ij3}^v) )</td>
<td>( (C_{hkl1}^v, C_{hkl2}^v, C_{hkl3}^v) )</td>
<td>( (FC_{v1}, FC_{v2}, FC_{v3}) )</td>
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<tr>
<td>Mutation strategy</td>
<td>Operator</td>
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<td>-----------------------</td>
<td>-----------------------------------------------</td>
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<td></td>
</tr>
<tr>
<td>Rand/1</td>
<td>( MV_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Best/1</td>
<td>( MV_i = \omega_{\text{best}} + F(\omega_{v1} - \omega_{v2}) )</td>
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<td></td>
</tr>
<tr>
<td>Current to best/1</td>
<td>( MV_i = \omega_{v1} + F(\omega_{\text{best}} - \omega_{i}) + F(\omega_{v1} - \omega_{v2}) )</td>
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<tr>
<td>Best/2</td>
<td>( MV_i = \omega_{\text{best}} + F(\omega_{v1} - \omega_{v2}) + F(\omega_{v3} - \omega_{v4}) )</td>
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<tr>
<td>Rand/2</td>
<td>( MV_i = \omega_{v1} + F(\omega_{v2} - \omega_{v3}) + F(\omega_{v4} - \omega_{v5}) )</td>
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Table 4. OFVs vs. alteration of $\alpha$ ($\varphi = (0.5,0.2,0.3), \vartheta = 0.5$).

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<tr>
<th>$\alpha$</th>
<th>Objective functions (UAM, LAM)</th>
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<th>$Z_3$</th>
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<td>(17634.40,18430.66)</td>
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<td>(18422.81,19966.06)</td>
<td>(104.13,110.46)</td>
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<td>(21400.21,22176.39)</td>
<td>(116.81,124.04)</td>
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<td>(272135.87,384439.74)</td>
<td>(23195.16,24243.40)</td>
<td>(125.84,127.04)</td>
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Table 5. OFVs vs. alteration of $\varphi_k$ ($\theta = 0.5$, $\alpha = 0.3$).

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<tr>
<th>$\varphi_k$</th>
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<th>Z_3</th>
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<td>Z_1</td>
<td>(18422.81,19966.06)</td>
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<td>(104.13,110.46)</td>
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<tr>
<td>(1/3,1/3,1/3)</td>
<td>(261797.74,341005.43)</td>
<td>Z_1</td>
<td>(18144.46,19401.69)</td>
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<td>(102.21,108.09)</td>
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<tr>
<td>(0.2,0.5,0.2)</td>
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<td>(109.70,115.74)</td>
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</table>
Table 6. OFVs vs. alteration of $\theta$ ($\varphi_r = (1/3,1/3,1/3), \alpha = 0.3$).

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<th>$Z_1$</th>
<th>$Z_2$</th>
<th>$Z_3$</th>
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<td>0.1</td>
<td>(275751.24,352132.37)</td>
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<td>(108.04,118.09)</td>
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<td>0.5</td>
<td>(261797.74,341005.43)</td>
<td>(18144.46,19401.69)</td>
<td>(102.21,108.09)</td>
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<td>0.9</td>
<td>(260100.11,337732.32)</td>
<td>(16231.92,18782.75)</td>
<td>(99.01,98.74)</td>
</tr>
</tbody>
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Table 7. HDE and HICA parameters settings.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Parameter</th>
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<td>HDE</td>
<td>Population size</td>
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<td>$F$</td>
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<td>Assimilation rate</td>
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<td>Iteration</td>
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<tr>
<td>HICA</td>
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<td>Revolution rate</td>
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<tr>
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<td>I</td>
<td>P</td>
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<tr>
<td>----------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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</table>
Table 9. Experimental results on HICA for small-sized test problems ($\alpha = 0.3$).

<table>
<thead>
<tr>
<th>Data set</th>
<th>$I$</th>
<th>$P$</th>
<th>CR(%)</th>
<th>ACR(%)</th>
<th>CD(%)</th>
<th>Time(s)</th>
</tr>
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<td></td>
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<td>1</td>
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<td>97.80</td>
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### Table 10. Results of the normality analysis.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Metric</th>
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<th>CD</th>
<th>Time</th>
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<tr>
<td>HDE</td>
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<td>0.091</td>
<td>0.2</td>
</tr>
<tr>
<td>HICA</td>
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<td><strong>0.002</strong></td>
<td>0.2</td>
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Table 11. Results of the heteroscedasticity analysis.

<table>
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<tbody>
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<td>P-value</td>
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<td><strong>0.021</strong></td>
<td>0.831</td>
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Table 12. Results of the paired t-test.

<table>
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<th>Metric</th>
<th>Paired Differences</th>
<th>95% Confidence Interval of the t</th>
<th>df</th>
<th>Sig. (2-tailed)</th>
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</thead>
<tbody>
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<td>Std. Deviation</td>
<td>Std. Error Mean</td>
<td>Lower</td>
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<td>ACR</td>
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<td>0.547</td>
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**Table 13. Results of the Wilcoxon test.**

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Table 14. APRG of HDE in comparison to HICA and computation time for large-sized test problems ($a = 0.3$).

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<th>$P$</th>
<th>Time(s)</th>
<th>Replications (HICA)</th>
<th>APRG(%)</th>
<th>Time(s)</th>
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